1 Ghost Track Algorithm

The B decay flavour tag with the charge dipole relies on the kinematic fact that the boost of the B decay system carries the cascade charm decay downstream from the B decay vertex. Monte Carlo studies show that in B decays producing a single D meson the cascade D decays on average 4200μm from the IP, while the intermediate B vertex is displaced on average only 46μm transversely from the line joining the IP to the D decay vertex. This kinematic stretching of the B decay chain into an approximately straight line is exploited by the ghost track algorithm. This new algorithm has two stages and operates on a given set of selected tracks in a jet or hemisphere. First, the best estimate of the straight line from the IP directed along the B decay chain is found. This line is promoted to the status of a track by assigning it a finite width. This new track, regarded as the resurrected image of the deceased B hadron, is called the 'ghost' track. Secondly, the selected tracks are vertexed with the ghost track and the IP to build up the decay chain along the ghost direction. Both stages are now described in more detail.

Given a set of tracks in a hadronic jet or hemisphere a new track G is created with the properties that it is a straight line from the IP directed along the jet or thrust axis and has a constant resolution width of 25μm in both $r\phi$ and rz. For each track i a vertex is formed with track G and the vertex location $\mathbf{r_i}$, fit χ^2_i and L_i are determined (L_i is the longitudinal displacement from the IP of $\mathbf{r_i}$ projected onto the direction of track G). This is calculated for each of the tracks and the summed χ^2 is formed:

$$\chi^2_S = \sum_i \begin{array}{ll} \chi^2_i & \mathrm{L}_i \geq 0.0 \\ (2\chi^2_{0i} - \chi^2_i) & \mathrm{L}_i < 0.0 \end{array} \qquad (1)$$

where χ^2_{0i} is the χ^2_i of track i to track G determined at $\mathrm{L}_i = 0$ rather than at the best fit vertex location. The aim is to construct this quantity, χ^2_S, such that when the direction of G is varied the minimum of χ^2_S provides the best estimate of the B decay direction. If the initial direction is a relatively long way from the B line of flight, some or all of the decay tracks may vertex with G with a negative value of L_i. In this case the $2\chi^2_{0i} - \chi^2_i$ term above helps to push track G towards the B flight path as χ^2_S is minimised. This first minimisation using equation 1 is designed for this purpose. (Note that the contribution of each track as χ^2_S is minimised changes in a continuous manner even if L_i changes sign since $\chi^2_i = \chi^2_{0i}$ at $\mathrm{L}_i = 0$).

The value of χ^2_S is recalculated as track G is rotated (about the pivot at the IP) incrementally in ever decreasing angular steps $\delta\theta$ and $\delta\phi$ until the minimum is found within the required precision (< 0.1mrad, i.e. within 1μm at 1cm from the IP). The width of track G is set such that the maximum $\chi^2_i = 1.0$ for all tracks with $\mathrm{L}_i > 0$. (if this is less than 25μm, it is restored to 25μm). The track G is now consistent with all potential B decay candidate tracks ($\mathrm{L}_i > 0$) at the level $\chi^2_i \leq 1.0$. In other words, the new width of G measures the degree to which the tracks conform to a straight line decay chain. A second iteration in $\delta\theta$,$\delta\phi$ now takes place with the summed χ^2 redefined as:

$$\chi^2_S = \sum_i \begin{array}{ll} \chi^2_i & \mathrm{L}_i \geq 0.0 \\ \chi^2_{0i} & \mathrm{L}_i < 0.0 \end{array} \qquad (2)$$

which is not sensitive to any spurious background track with a negative value of L_i which might otherwise perturb the direction of track G. After finding the new minimum of

χ^2_S the width of G is again recalculated such that $\chi^2_i \leq 1.0$ for all tracks i with $L_i > 0$. Again this width is required to be at least 25μm. Track G is now directed along the best guess of the B decay line of flight and has a width such that it is consistent with potential B decay tracks in the jet, track G is now called the 'ghost' track.

The second stage of the algorithm begins by defining a fit probability for a set of tracks to form a vertex with each other and with the ghost track (or IP). This probability then measures the likelihood of the set of tracks both belonging to a common vertex *and* being consistent with the ghost track (or IP) and hence forming a part of the B decay chain. These probabilities are determined from the fit χ^2 which is in turn determined algebraically from the parameters of the selected tracks and the ghost track (or the $7 \times 7 \times 30\mu$m ellipsoid assumed for the IP). The earlier requirement that each $L_i > 0$ track makes a $\chi^2_i \leq 1.0$ with the ghost track has the effect that the fit probabilities have the desired property of having an approximately flat distribution from 0.0 to 1.0 for genuine vertices, independent of both multiplicity and decay length. This property also relies on the choice of the number of degrees of freedom as 2N$-$2 (or 2N) when fitting N tracks together with the ghost track (or IP). Fake vertices peak at probability close to 0.0.

For a set of N tracks, there are initially N+1 candidate vertices (N 1-prong secondary vertices and a bare IP). A matrix of track i – track j associations is constructed to store the calculated probabilities of each candidate vertex pair fitted together with the ghost track. A further column and row is added to the matrix to store the probabilities of each track fit with the IP ellipsoid. The upper triangle of the matrix (i.e. the ij $(i < j)$ elements) stores the probabilities while the lower triangle (initialised with ij $(i > j)$ elements set to 0.0) indelibly records which tracks (and IP) have been assigned together in a common vertices as the algorithm progresses. Once the upper triangle has been filled, the highest probability in the matrix table is found and the corresponding candidate vertex pair are from then on tied together in a new candidate vertex for all future computations by flagging the corresponding lower triangle elements of the matrix with non-zero values. The upper triangle of the matrix is now refilled taking into account the associations that have so far been made, the new maximum probability is found, and the corresponding subset of the tracks and IP is tied together. At each iteration of combining the maximum probability matrix element contributors, the number of candidate vertices decreases by one. The iterations continue until the maximum probability is less than 1%. At this point the tracks and IP have been divided into unique subsets by the associations thereby defining topological vertices.

Jets or hemispheres in which three vertices are found – the primary (which includes by definition the IP), a secondary and a tertiary – are used for the charge dipole analysis. The secondary vertex is identified as the B decay vertex and the tertiary as the cascade charm decay. As well as well improving the purity and efficiency of the dipole reconstruction (by requiring the vertices be consistent with a single line of flight) the ghost track algorithm has the additional advantage of allowing the direct reconstruction of 1-prong vertices, including the topology consisting of 1-prong B decay and D decays.